Int. J. Heat Mass Transfer. Vol. 34, No. 1, pp. 79–85, 1991
Printed in Great Britain

0017-9310/91 \$3.00+0.00

Two-dimensional rectangular fin with variable heat transfer coefficient

S. W. MA,† A. I. BEHBAHANI‡ and Y. G. TSUEI†

† Department of Mechanical and Industrial Engineering, University of Cincinnati, OH 45221-0072, U.S.A.
‡ Energy Research Inc., P.O. Box 2034, Rockville, MD 20852, U.S.A.

(*Received* 4 *October* 1989 *and in final form* 30 *January* 1990)

Abstract—A Fourier series approach is used to investigate a two-dimensional rectangular fin with arbitrary variable heat transfer coefficient on the fin surface. The solutions for temperature distribution with three different boundary conditions at the fin tip have been obtained. Based on the uniqueness principle of practical problems, these solutions will lead to the familiar expressions when the heat transfer coefficient is constant although they are different in form.

1. INTRODUCTION

MANY FIN problems have been solved under the assumption of constant heat transfer coefficient along the entire length of the fin. A variety of results can be found in ref. [1]. Unfortunately, in the design process of the fin this assumption is not quite valid [2]. For example, with regard to the effect of variability of heat transfer coefficient on the single material fin, finite difference calculations with a linearly increasing value of the coefficient showed a 39% reduction of heat transfer in some cases [3]. Theoretically, it is shown that for a flat fin the effects of the leading edge, the boundary layer, and turbulence can lead to marked variations in the heat transfer coefficient. Obviously, an analysis of the combined conduction–convection system with a variable heat transfer coefficient is important from a practical viewpoint.

Historically, Han and Lefkowitz [4] studied a one-dimensional longitudinal fin of rectangular profile with arbitrary heat transfer coefficient, $h(x)$, taken as a power series of the distance x from the fin base. Chen and Zyskowski [5] took the same problem with an exponential variation of the heat transfer coefficient, while Heggs *et al.* [6] considered the temperature distribution within an annular triangular fin with heat transfer coefficients increased linearly from the fin base to the fin tip. These investigations showed that, in some cases, the assumed variations can give results which are in excellent agreement with those obtained experimentally. However, for any given problem the appropriate variation cannot be prescribed by these simple functions and consequently uniform heat transfer coefficients are still used. More recently, Snider and Kraus [7] developed an expression for the average heat transfer coefficient which can be determined by use of a lengthwise weighting function $w(x)$. Ünal [8, 9] studied a one-dimensional fin with heat generation and non-uniform heat transfer coefficient which is a function of temperature. Barrow [10] presented a series solution for the one-dimensional temperature distribution along a rectangular fin with variable surface heat transfer coefficient.

All of the previous studies are based on the one-dimensional fin. Some researchers [11–13] reported that the one-dimensional analysis can be used instead of two-dimensional analysis only if the transverse Biot number, *Bi*, is much less than unity. Suryanarayana [12] has revealed that the fin heat fluxes can be lower by 80% than those predicted by the one-dimensional approach in some cases. Thus, the analysis for the two-dimensional fin with variable heat transfer coefficients is important in theory and practice. Barrow *et al.* [3] predicted the temperature and heat flow in rectangular composite fins with a linearly increasing distribution of heat transfer coefficients by finite element and finite difference methods. However, the analytical solution of this problem has not been obtained because the boundary conditions become complicated. Mathematically, difficulty arises for the Sturm–Liouville problem with non-constant heat transfer coefficient boundary condition according to the traditional approach (separation of variables). This paper develops a Fourier series approach to solve the two-dimensional rectangular fin. The temperature distribution in the rectangular fin with arbitrary variable heat transfer coefficient has been written in terms of a summation of series. The analytical solution has been described for the basic governing equation with heat dissipation by convection from the tip of the fin to the surroundings at T_{∞}. In Section 3, solutions for two other boundaries of the tip of a fin in which the tip temperature is equal to that of the surroundings and the tip is insulated are derived.

2. SOLUTION UNDER CONSTANT TIP SURFACE CONVECTION COEFFICIENT

Consider a rectangular fin ($2B \times L$) where both the upper and the lower surfaces dissipate heat by con-

NOMENCLATURE

a_n, b_n	dimensionless unknown constants
b_{mj}, b^*_{mj}	dimensionless elements in matrix
B	half thickness of fin [m]
Bi	transverse Biot number, hB/k
c_m	dimensionless unknown constants
D	dimensionless constant
h, $h(x)$	heat transfer coefficient [W m^{-2} K^{-1}]
H	ratio of heat transfer coefficient and thermal conductivity [m^{-1}]
k	thermal conductivity of fin material [W m^{-1} K^{-1}]
L	fin length [m]
P	fin perimeter [m]
r_m	dimensionless elements in column matrix
T	temperature [K]
T_∞	temperature of surroundings [K]
x, y	Cartesian coordinates [m]
W	width of fin [m].

Greek symbols

β	roots of transcendental equations [m^{-1}]
μ	thermal transmission ratio [W K^{-1}]
ϕ, Φ	dimensionless temperature.

Subscripts

b	base of fin
c	constant
j, m, n	indexes
L	tip of fin.

vection to a surrounding at temperature, T_∞, with variable heat transfer coefficient $h(x)$ (Fig. 1). Furthermore, the tip of the fin has a constant heat transfer coefficient h_L and the temperature of the fin base is T_b. Under steady state condition and no heat generation, the governing equation for the two-dimensional fin of the homogeneous material is

$$\frac{\partial^2\phi(x,y)}{\partial x^2}+\frac{\partial^2\phi(x,y)}{\partial y^2}=0 \qquad (1)$$

where $\phi(x, y)$ is a dimensionless temperature

$$\phi(x,y)=\frac{T-T_\infty}{T_b-T_\infty} \qquad (2)$$

with the following boundary conditions:

$$\phi=1 \qquad x=0,\quad 0\leqslant y\leqslant B; \qquad (3)$$

$$-\frac{\partial\phi}{\partial x}=\frac{h_L}{k}\phi \qquad x=L,\quad 0\leqslant y\leqslant B; \qquad (4)$$

$$\frac{\partial\phi}{\partial y}=0 \qquad y=0,\quad 0\leqslant x\leqslant L; \qquad (5)$$

$$-\frac{\partial\phi}{\partial y}=\frac{h(x)}{k}\phi \qquad y=B,\quad 0\leqslant x\leqslant L; \qquad (6)$$

where k is the conductivity of the fin material. By use of linear combination of the Fourier cosine series about y and the sine series about x [14], the solution which satisfies the governing equation (1) can be assumed as

$$\phi=\sum_{n=1,3}^{\infty}\left[a_n\cosh\frac{n\pi x}{2B}+b_n\sinh\frac{n\pi x}{2B}\right]\cos\frac{n\pi y}{2B}+\sum_{m=1}^{\infty}c_m\cosh\beta_m y\sin\beta_m x \qquad (7)$$

where the four undetermined constants a_n, b_n, c_m and β_m are not independent. Their relations can be found through the boundary conditions. However, the

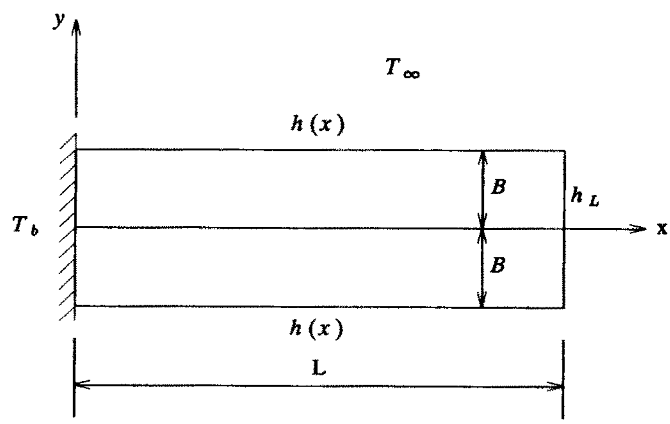

FIG. 1. Geometry of a two-dimensional fin.

above solution satisfies boundary (5) automatically. It is obvious that boundary condition (4) must be split into two parts so that two sets of constants can be determined. The other boundary conditions (3) and (6) can be used to determine the remainder of the constants. First, introducing solution (7) into boundary condition (3) the following expression is obtained:

$$\sum_{n=1,3}^{\infty} a_n \cos\frac{n\pi y}{2B} = 1. \quad (8)$$

By the inversion of the Fourier transform, the constant a_n can be found in terms of

$$a_n = \frac{4(-1)^{(n-1)/2}}{n\pi}. \quad (9)$$

After substituting equation (7) into boundary condition (4) we have

$$-\sum_{n=1,3}^{\infty} \frac{n\pi}{2B}\left[a_n \sinh\frac{n\pi L}{2B} + b_n \cosh\frac{n\pi L}{2B}\right]\cos\frac{n\pi y}{2B}$$
$$-\sum_{m=1}^{\infty} c_m\beta_m \cosh\beta_m y \cos\beta_m L$$
$$= \frac{h_L}{k}\left\{\sum_{n=1,3}^{\infty}\left[a_n \cosh\frac{n\pi L}{2B} + b_n \sinh\frac{n\pi L}{2B}\right]\cos\frac{n\pi y}{2B}\right.$$
$$\left. + \sum_{m=1}^{\infty} c_m \cosh\beta_m y \sin\beta_m L\right\}. \quad (10)$$

As mentioned above, expression (10) has to be divided into two parts; i.e. from the second terms on both sides, the eigenvalues β_m can be written as

$$\beta_m L \cot\beta_m L = -\frac{h_L L}{k}. \quad (11)$$

From the first terms on both sides of expression (10) b_n can be expressed in terms of a_n

$$b_n = -a_n D_n \quad (12)$$

where D_n stands for

$$D_n = \frac{\dfrac{n\pi L}{2B}\tanh\dfrac{n\pi L}{2B} + \dfrac{h_L L}{k}}{\dfrac{n\pi L}{2B} + \dfrac{h_L L}{k}\tanh\dfrac{n\pi L}{2B}}. \quad (13)$$

Finally, c_m can be found by considering boundary condition (6). Introducing solution (7) into boundary condition (6), we have

$$\sum_{m=1}^{\infty} c_m\beta_m \sinh\beta_m B \sin\beta_m x$$
$$+\frac{h(x)}{k}\sum_{m=1}^{\infty} c_m \cosh\beta_m B \sin\beta_m x$$
$$= \sum_{n=1,3}^{\infty}\frac{n\pi}{2B}(-1)^{(n-1)/2}\left[a_n \cosh\frac{n\pi x}{2B} + b_n \sinh\frac{n\pi x}{2B}\right]. \quad (14)$$

Utilizing the inversion formula of the Fourier transform over the region $(0, L)$, noting expressions (9) and (12) and making suitable changes for some subscripts, we find

$$c_m b_m^* + \sum_{j=1}^{\infty} c_j b_{mj}^* = r_m \quad (m = 1, 2, \ldots) \quad (15)$$

where

$$\left.\begin{aligned} b_m^* &= 0.5\sinh\beta_m B(\beta_m L - \sin\beta_m L\cos\beta_m L) \\ b_{mj}^* &= \cosh\beta_j B\int_0^L \frac{h(x)}{k}\sin\beta_j x \sin\beta_m x\,dx \\ r_m &= \sum_{n=1,3}^{\infty}\frac{2}{B}\int_0^L\left[\cosh\frac{n\pi x}{2B}\right. \\ &\qquad \left. - D_n\sinh\frac{n\pi x}{2B}\right]\sin\beta_m x\,dx \end{aligned}\right\} \quad (16)$$

where D_n is expressed by equation (14). c_m in equation (15) can be found by

$$\begin{bmatrix} b_{11} & b_{12} & \cdot & \cdot & b_{1j} & \cdot \\ b_{21} & b_{22} & \cdot & \cdot & b_{2j} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ b_{m1} & b_{m2} & \cdot & \cdot & b_{mj} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ \cdot \\ c_m \\ \cdot \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ \cdot \\ r_m \\ \cdot \end{bmatrix} \quad (17)$$

where

$$b_{mj} = \begin{cases} b_{mj}^* & \text{if } j \neq m \\ b_m^* + b_{mm}^* & \text{if } j = m. \end{cases} \quad (18)$$

Therefore, solution (7) can be expressed by

$$\phi(x, y) = \sum_{n=1,3}^{\infty}\frac{(-1)^{(n-1)/2}}{\dfrac{n\pi}{4}}$$
$$\times\frac{\dfrac{n\pi L}{2B}\cosh\dfrac{n\pi(L-x)}{2B} + \dfrac{h_L L}{2B}\sinh\dfrac{n\pi(L-x)}{2B}}{\dfrac{n\pi L}{2B}\cosh\dfrac{n\pi L}{2B} + \dfrac{h_L L}{2B}\sinh\dfrac{n\pi L}{2B}}\cos\frac{n\pi y}{2B}$$
$$+\sum_{m=1}^{\infty} c_m \cosh\beta_m y \sin\beta_m x \quad (19)$$

where β_m and c_m are determined by equations (11) and (17), respectively.

3. SOLUTIONS FOR THE TIP BOUNDARY CONDITIONS OF THE FIRST AND SECOND KINDS

The solutions in the two cases can be obtained directly from that of Section 2 by approximation [1]. In the first case $h_L \to \infty$, the temperature of the fin tip is that of the surroundings, T_∞, hence $\phi(L, y)$ is equal to zero. In the second case $h_L \to 0$, the fin tip is insulated. The analytical process can be, respectively, shown as follows.

Case 1

For $h_l \to \infty$, equation (11) can be simplified as

$$\sin \beta_m L = 0. \tag{20}$$

Thus the roots of the transcendental equation are

$$\beta_m = \frac{m\pi}{L} \quad (m = 1, 2, \ldots) \tag{21}$$

and equation (13) becomes

$$D_n = \coth \frac{n\pi L}{2B}. \tag{22}$$

With the aid of the formula [15]

$$\tanh \frac{\pi x}{2} = \frac{4x}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2 + x^2} \tag{23}$$

equation (16) is written as

$$\left.\begin{aligned} b_m^* &= \frac{m\pi}{2} \sinh \frac{m\pi B}{L} \\ b_{mj}^* &= \cosh \frac{j\pi B}{L} \int_0^L \frac{h(x)}{k} \sin \frac{j\pi x}{L} \sin \frac{m\pi x}{L} \, dx \\ r_m &= \tanh \frac{m\pi B}{L}. \end{aligned}\right\} \tag{24}$$

In this case, solution (19) can be expressed by

$$\phi(x, y) = \sum_{n=1,3}^{\infty} \frac{(-1)^{(n-1)/2}}{\frac{n\pi}{4}} \cdot \frac{\sinh \frac{n\pi(L-x)}{2B}}{\sinh \frac{n\pi L}{2B}} \cos \frac{n\pi y}{2B} + \sum_{m=1}^{\infty} c_m \cosh \frac{m\pi y}{L} \sin \frac{m\pi x}{L} \tag{25}$$

where c_m can be determined from equation (17). When $h(x)$ is equal to a constant h_c, the integral corresponding to b_{mj}^* in equation (24) satisfies the orthogonal condition, which can be expressed in terms of

$$b_{mj}^* = \begin{cases} 0 & \text{if } j \neq m \\ \frac{h_c L}{2k} \cosh \frac{m\pi B}{L} & \text{if } j = m. \end{cases} \tag{26}$$

Substituting equations (26) and (24) into equation (18), with the aid of equation (17) c_m can be found at once

$$c_m = \frac{2 \tanh \frac{m\pi B}{L}}{m\pi \sinh \frac{m\pi B}{L} + \frac{h_c L}{k} \cosh \frac{m\pi B}{2L}}. \tag{27}$$

Therefore, when $h(x)$ is equal to a constant h_c, the following closed form solution is obtained:

$$\phi(x, y) = \sum_{n=1,3}^{\infty} \frac{(-1)^{(n-1)/2} \sinh \frac{n\pi(L-x)}{2B}}{\frac{n\pi}{4} \sinh \frac{n\pi L}{2B}} \cos \frac{n\pi y}{2B} + \sum_{m=1}^{\infty} \frac{2 \tanh \frac{m\pi B}{L} \cosh \frac{m\pi y}{L} \sin \frac{m\pi x}{L}}{m\pi \sinh \frac{m\pi B}{L} + \frac{h_c L}{k} \cosh \frac{m\pi B}{L}}. \tag{28}$$

Case 2

For $h_L \to 0$, equation (11) can be simplified as

$$\cos \beta_m L = 0. \tag{29}$$

The roots of the above transcendental equation are

$$\beta_m = \frac{(2m-1)\pi}{2L} \quad \text{for} \quad m = 1, 2, \ldots \tag{30}$$

and equation (13) becomes

$$D_n = \tanh \frac{n\pi L}{2B}. \tag{31}$$

By use of equation (23), equation (16) is written as

$$\begin{aligned} b_m^* &= \frac{(2m-1)\pi}{4} \sinh \frac{(2m-1)\pi B}{2L} \\ b_{mj}^* &= \cosh \frac{(2j-1)\pi B}{2L} \int_0^L \frac{h(x)}{k} \sin \frac{(2j-1)\pi x}{2L} \\ &\quad \times \sin \frac{(2m-1)\pi x}{2L} \, dx \\ r_m &= \tanh \frac{(2m-1)\pi B}{2L}. \end{aligned} \tag{32}$$

Solution (19) can be expressed by

$$\phi(x, y) = \sum_{n=1,3}^{\infty} \frac{(-1)^{(n-1)/2}}{\frac{n\pi}{4}} \cdot \frac{\cosh \frac{n\pi(L-x)}{2B}}{\cosh \frac{n\pi L}{2B}} \cos \frac{n\pi y}{2B} + \sum_{m=1}^{\infty} c_m \cosh \frac{(2m-1)\pi y}{2L} \sin \frac{(2m-1)\pi x}{2L} \tag{33}$$

where c_m can be found by equation (17). If $h(x)$ is equal to a constant h_c, b_{mj}^* in equation (32) can be further reduced

$$b_{mj}^* = \begin{cases} 0 & \text{if } j \neq m \\ \frac{h_c L}{2k} \cosh \frac{(2m-1)\pi B}{2L} & \text{if } j = m. \end{cases} \tag{34}$$

Similarly

$$c_m = \frac{2 \tanh \frac{(2m-1)\pi}{2L}}{\frac{(2m-1)\pi}{2} \sinh \frac{(2m-1)\pi B}{2L} + \frac{h_c L}{k} \cosh \frac{(2m-1)\pi B}{2L}}. \tag{35}$$

In this case solution (33) has a closed form

$$\phi(x,y) = \sum_{n=1,3}^{\infty} \left[\frac{(-1)^{(n-1)/2} \cosh \frac{n\pi(L-x)}{2B} \cos \frac{n\pi y}{2B}}{\frac{n\pi}{4} \cosh \frac{n\pi L}{2B}} + \frac{2 \tanh \frac{n\pi B}{2L} \cosh \frac{n\pi y}{2L} \sin \frac{n\pi x}{2L}}{\frac{n\pi}{2} \sinh \frac{n\pi B}{2L} + \frac{h_c L}{k} \cosh \frac{n\pi B}{2L}} \right]. \quad (36)$$

As will be discussed later solution (36) does not require the calculation of eigenvalues and eigenfunctions.

4. DISCUSSION AND A PRACTICAL EXAMPLE

Under the boundary condition of the previous section (i.e. $h_L \to \infty$ and $h_L \to 0$) and $h(x)$ is equal to a constant, h_c, the forms of solutions (28) and (36) are different from the familiar expressions in most references where the expressions are given in terms of eigenvalues. For example, the solution in case 2 is expressed by [1]

$$\phi(x,y) = \sum_{j=1}^{\infty} \frac{2H \cosh \beta_j(L-x) \cos \beta_j y}{[B(\beta_j^2+H^2)+H] \cosh \beta_j L \cos \beta_j B} \quad (37)$$

where H is defined by h_c/k and β_j are the roots of the transcendental equation

$$\beta_j \tan \beta_j B = H \equiv \frac{h_c}{k}. \quad (38)$$

Solution (37) is derived from the basic governing differential equation (1) and the following boundary conditions:

$$\phi = 1 \qquad x=0, \quad 0 \leqslant y \leqslant B; \quad (39)$$

$$-\frac{\partial \phi}{\partial x} = 0 \qquad x=L, \quad 0 \leqslant y \leqslant B; \quad (40)$$

$$\frac{\partial \phi}{\partial y} = 0 \qquad y=0, \quad 0 \leqslant x \leqslant L; \quad (41)$$

$$-\frac{\partial \phi}{\partial y} = \frac{h_c}{k}\phi \qquad y=B, \quad 0 \leqslant x \leqslant L. \quad (42)$$

It is not difficult to show that solution (36) of this paper also satisfies equation (1) and the above boundary conditions. Based on the uniqueness principle of this problem, the dimensionless temperature (36) is identical with equation (37) even though they are different in form. Since our solution (36) has been written in a form that does not require eigenfunctions, it is suitable for a calculator. It should be noted that the series in equation (36) converges with order one, which is slower than that in equation (37). Thus the classical solution is still a better method for the calculation of physical quantities near the boundaries of the fin such as temperature, heat flow, etc.

It has been shown that the proposed approach can be used not only in the problems where $h(x)$ is continuous in the x-direction but also in non-continuous problems as long as $h(x)$ satisfies the Dirichlet conditions which mean that $h(x)$ is single valued, finite, and sectionally continuous and cannot have an infinite number of maxima and minima in the interval (0, L). As an example where $h(x)$ is discontinuous, a square spine [6] is considered as shown in Fig. 2. The fin thermal conductivity k, is taken to be 100 W m^{-1} K^{-1}. The heat transfer coefficient $h(x)$ is assumed to be 50 W m^{-2} K^{-1} along the first 0.01 m of its length, and 100 W m^{-2} K^{-1} along the remaining 0.03 m. This situation will produce a boundary layer along the base surface inhibiting the heat transfer [6]. The typical temperature curves ($\phi(x, 0)$ and $\phi(x, B)$) have been plotted in Fig. 3. The thermal transmission ratio μ, which is the ratio of the rate of heat dissipated by the fin to the temperature difference at the base [16], is given by

$$\mu = P \int_0^L h(x)\phi(x,B)\,dx \quad (43)$$

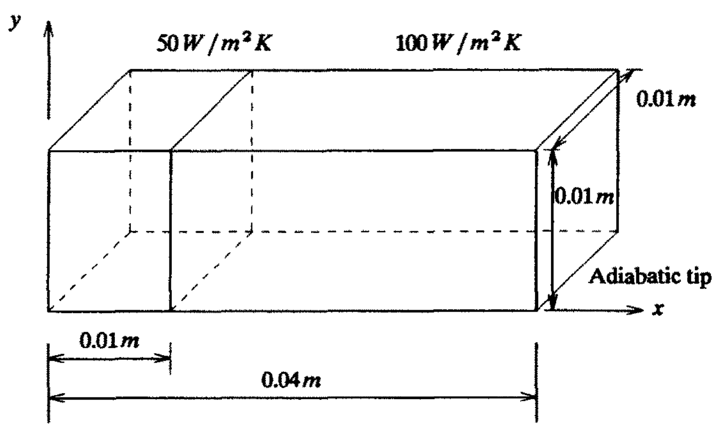

FIG. 2. Fin with non-continuous heat transfer coefficients.

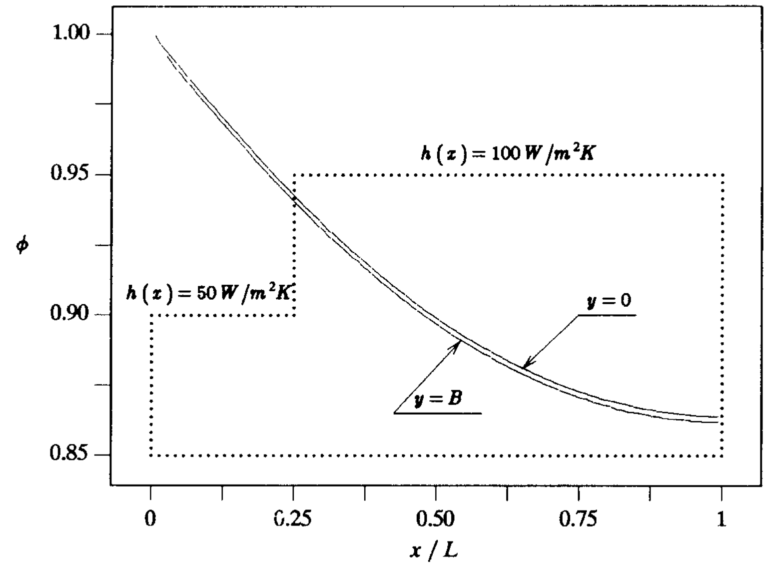

FIG. 3. Graph of dimensionless temperature.

where P is fin perimeter. Under the condition of an adiabatic tip (case 2), with equation (33) the thermal transmission ratio μ is given in terms of a series

$$\mu = 4W \sum_{m=1}^{I} c_m \cosh\frac{(2m-1)\pi B}{2L} \times \int_0^L h(x)\sin\frac{(2m-1)\pi x}{2L}\,\mathrm{d}x \quad (44)$$

which, when evaluated numerically, leads to

$$\mu = 0.1259\ \mathrm{W\,K^{-1}}. \quad (45)$$

For comparison, μ can be computed exactly by regarding the spine as two fins in cascade and the one-dimensional method [16]. The result is

$$\mu = 0.1150\ \mathrm{W\,K^{-1}} \quad (46)$$

an error of 8.7% on the above two-dimensional analysis (45). Several works [3, 17] have shown that the solution for two-dimensional heat flow in a fin with a constant heat transfer coefficient is less than that predicted by the one-dimensional approach. However, it is interesting to note that our result is greater than that from the one-dimensional method since the heat transfer coefficient is not constant. Recalling all the derivative processes, the result in this investigation is based on an assumption that the base temperature, T_b, of the fin is constant. The effects of a non-constant fin base temperature have received some attention [10, 18]. However, it is not difficult to expand the presented method into those cases provided the boundary condition expressed in equation (1) is changed suitably so that the expression of a_n is different from that of equation (9).

As mentioned above, our solutions converge with order one (equations (19), (25) and (33)). Furthermore it is seen that on the surface of a fin it takes a longer time to solve the set of linear algebraic equations (17) due to slower convergence. This is a common phenomenon for most finite two-dimensional, steady heat conduction problems with no internal heat generation [17–19]. Although ref. [1] did manage to utilize the transcendental equation (38) for solution (37) with convergence of order two, it is for the case of constant heat transfer coefficient.

5. CONCLUSION

The technique of linearly combining functions to form a general series is used to investigate a two-dimensional rectangular fin for three boundary conditions at the fin tip. The results of the temperature distribution include the situation when a heat transfer coefficient is constant. Based on the uniform distribution of temperature at the fin base, the difference between prediction of one-dimensional computation and that of two-dimensional analysis is about 8.7%. However, it is not difficult to expand the presented method into the cases of a non-constant fin base temperature. It is noted that the method is applied to arbitrary heat transfer coefficients even though they are discontinuous.

REFERENCES

1. M. N. Özisik, *Heat Conduction*. Wiley, New York (1980).
2. M. L. Ghai, Heat transfer in straight fins. In *General Discussion on Heat Transfer*, pp. 180–182. ASME (1951).
3. H. Barrow, J. Mistry and D. Clayton, Numerical and exact mathematical analyses of two-dimensional rectangular composite fins, *8th Int. Heat Transfer Conf.*, San Francisco, Vol. 2, pp. 367–372 (1986).

4. L. S. Han and S. G. Lefkowitz, Constant cross-section fin efficiencies for nonuniform surface heat transfer coefficient, ASME Paper 60-WA-41 (1960).
5. S. Y. Chen and G. L. Zyskowski, Steady-state heat conduction in a straight fin with variable film coefficient, ASME Paper 63-HT-12 (1963).
6. P. J. Heggs, D. B. Ingham and M. Manzoor, The effects of non-uniform heat transfer from an annular fin of triangular profile, *ASME J. Heat Transfer* **103**, 184–185 (1981).
7. A. D. Snider and A. D. Kraus, Correcting for the variability of the heat transfer coefficient in extended surface analysis, *7th Int. Heat Transfer Conf.*, Munich, Vol. 6, pp. 239–243 (1982).
8. H. C. Ünal, Determination of the temperature distribution in an extended surface with a non-uniform heat transfer coefficient, *Int. J. Heat Mass Transfer* **28**, 2279–2284 (1985).
9. H. C. Ünal, Temperature distributions in fins with uniform and non-uniform heat generation and non-uniform heat transfer coefficient, *Int. J. Heat Mass Transfer* **30**, 1465–1477 (1987).
10. H. Barrow, Theoretical solution for the temperature in a straight fin with variable surface heat transfer coefficient, *J. Heat Recovery Syst.* **6**, 465 (1986).
11. E. M. Sparrow and L. Lee, Effects of fin-base temperature depression in a multifin array, *ASME J. Heat Transfer* **97**, 463–465 (1975).
12. N. V. Suryanarayana, Two-dimensional effects on heat transfer rates from an array of straight fins, *ASME J. Heat Transfer* **99**, 129–132 (1977).
13. P. J. Heggs and P. R. Stones, The effects of dimensions on the heat flowrate through extended surface, *ASME J. Heat Transfer* **102**, 180–182 (1980).
14. R. V. Churchil and J. W. Brown, *Fourier Series and Boundary Value Problems*. McGraw-Hill, New York (1987).
15. I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, pp. 35–45. Academic Press, New York (1980).
16. A. D. Kraus, A. D. Snider and L. F. Doty, An efficient algorithm for evaluating average of extended surface, *ASME J. Heat Transfer* **100**, 288–290 (1978).
17. Wah Lau and C. W. Tan, Errors in one-dimensional heat transfer analysis in straight and annular fins, *ASME J. Heat Transfer* **93**, 549–551 (1973).
18. D. C. Look, Jr., Two-dimensional fin with non-constant root temperature, *Int. J. Heat Mass Transfer* **32**, 977–980 (1989).
19. V. S. Arpaci, *Conduction Heat Transfer*. Addison-Wesley, Reading, Massachusetts (1966).

AILETTE RECTANGULAIRE BIDIMENSIONNELLE AVEC COEFFICIENT DE TRANSFERT THERMIQUE VARIABLE

Résumé—On utilise une approche par série de Fourier pour étudier une ailette bidimensionnelle rectangulaire avec coefficient de transfert thermique variable sur la surface. Les solutions pour la distribution de température sont obtenues pour trois conditions aux limites différentes au sommet de l'ailette. A partir du principe d'unicité des problèmes pratiques, ces solutions conduisent à des expressions familières quand le coefficient de transfert est constant bien qu'elles aient des formes différentes.

BETRACHTUNG EINER ZWEIDIMENSIONALEN RECHTECKRIPPE MIT VARIABLEN WÄRMEÜBERGANGSKOEFFIZIENTEN

Zusammenfassung—Mit Hilfe eines Fourier-Reihenansatzes wird eine zweidimensionale Rechteckrippe mit beliebigen veränderlichen Wärmeübergangskoeffizienten an der Oberfläche untersucht. Für drei unterschiedliche Randbedingungen an der Rippenspitze wird die Temperaturverteilung berechnet. Aufgrund der Einzigartigkeit praktischer Probleme führen diese Lösungen zu ähnlichen Ausdrücken, sofern der Wärmeübergangskoeffizient konstant ist.

ДВУМЕРНОЕ ПРЯМОУГОЛЬНОЕ РЕБРО С ПЕРЕМЕННЫМ КОЭФФИЦИЕНТОМ ТЕПЛООТДАЧИ

Аннотация—С помощью рядов Фурье исследуется теплообмен двумерного прямоугольного ребра с произвольным переменным коэффициентом теплоотдачи на поверхности. Получены решения для распределения температур при трех различных граничных условиях на вершине ребра. В силу принципмальной общности практических задач эти решения приводят к известным, хотя и различным по форме, выражениям, полученным при постоянном коэффициенте теплоотдачи.